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The Motion of Rigid Bodies Shaped Rectangular-Box: Experiment and Numerical Simulation

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Abstract

The system of rectangular-box structures is an especially concerning problem in building with its useful applications. Conducting investigations into the contact of these blocks in the movement process plays an important role in predicting the destruction of infrastructures. For that reason, the research group focus on how the rectangular-box rigid bodies move (includes translational and rotational movement) and interact. Herein, the numerical method SPH (Smoothed Particle Hydrodynamics) has been applied and improved to model this problem. The simulation result of blocks was compared with the physical experiment in the same condition and showed satisfactory agreement between them. This suggests that the numerical approach is a suitable proposal to simulate movement behaviors of rigid bodies.

Keywords: Rectangular-box rigid bodies; Movement; Numerical method SPH; Experiment.

1. Introduction

The rectangular-box blocks appear in many fields of our lives, as containers are used for agricultural products, material boxes in industry, conveyance of goods in transport and popular applications in construction. The design of retaining wall contains structures in the form of rectangular-box shape is one of the usefulness and convenience solutions in disaster mitigation due to landslide. So, the motion or interaction of these rigid bodies caused by gravity should be concerned and researched.

Conducting an experiment or numerical computing the large deformation to model the destruction of buildings is necessary. However, the number of studies the movement process of rectangular-box rigid bodies have not been much interested and the results in this problem still have limits. The Finite Element Method (FEM) (Johansson, 2019) is a widespread numerical tool but it is suffered from grid distortions. So, it is difficult to simulate the movement of rigid bodies and predict the flexible behaviours of them. This approach is more suitable for modelling translational motion than rotational motion. For those reasons, the mesh-free methods are alternative solutions and applied in complicated problems. The Discrete Element Method (DEM) (Cundall, 1979; Liu, 2018) is a popular mesh-free approach in geotechnical field. However, it is not easy to select parameters for contract law. Besides, Discontinuous Deformation Analysis (DDA) (Shi, 1988) is also used in contact modelling between blocks but in discontinuous environmental condition like rock mechanics. Some other messless methods, Material Point Method (MPM) (Sulsky, 1994; Bandara, 2015), Particle in Cell (PIC) (Harlow, 1964), etc.. generally require a large number of computational resources, time-consuming and complicated modelling.

In this paper, Smoothed Particle Hydrodynamics (Gingold & Monaghan, 1977; Lucy, 1977) is a powerful numerical approach to simulate the large deformations and failure behaviours of materials or rectangular-box structures in retaining wall system. So, the physical experiment is used to compare the numerical result in simulating motion of blocks based on the SPH method.

2. Methodology

2.1. SPH approach

The Smoothed Particle Hydrodynamics (SPH) method, originally proposed by Gingold & Monaghan and Lucy (Gingold, 1977; Lucy, 1977) have applied and advanced in some publications of research group (Ha, 2011; Cuong, 2013; Cuong, 2015; Cuong, 2017). The SPH approximation for a function $A(r_a)$ at the position r_a of particle a and for a gradient term (Liu, 2003) may be respectively written as:

$$A(r_a) = \sum_{b=1}^{N} m_b \frac{A_b}{\rho_b} W_{ab} \tag{1}$$

$$\nabla A(r_a) = \sum_{b=1}^{N} m_b \frac{A_b}{\rho_b} \nabla_a W_{ab} \tag{2}$$

where the subscript b refers to the quantity evaluated at the position of particle b, N is the number of neighboring particles, m is the mass of particle b, ρ is the density of particle b, W is Kernel function.

2.2. Motion modelling of rigid bodies

In this paper, three individual rectangular blocks are simulated, each is assumed as a rigid body and has complete degrees of freedom (consider motion in two dimensions). The translational and rotational motion of the center point of a rigid body may be determined below.

The motion equation of block is given as follow:

$$M \frac{d\mathbf{V}}{dt} = \mathbf{F} \tag{3}$$

where M is the mass of body, $\mathbf{V}$ is the velocity vector, $\mathbf{F}$ is total force vector acting on the body.

The equation of rotation motion about the mass central of rigid body is:

$$I \frac{d\mathbf{\Omega}}{dt} = \boldsymbol{T} \tag{4}$$

where I is is the inertial moment, $\mathbf{\Omega}$ is the angular velocity, $\boldsymbol{T}$ is the total torque about the mass central.

The rigid body is represented by the set of boundary particles that are equi-spaced around the boundary. The force vector acting on each boundary particle i located on the moving block is f_i. Equation (1), (2) can be rewriten:

$$M \frac{d\boldsymbol{V}}{dt} = \sum_i \boldsymbol{f}_i \tag{5}$$

$$I \frac{d\mathbf{\Omega}}{dt} = \sum_i (\boldsymbol{r}_i - \boldsymbol{R}) \times \boldsymbol{f}_i \tag{6}$$

where $\boldsymbol{r_i}$ và $\boldsymbol{R}$ are vector coordinates of boundary particle and mass central, respectively. The boundary particles move as a part of the rigid body, thus the change on position of boundary particle *i* can be determined by the translational and rotational motion about the center of mass, is given by:

$$\frac{d\boldsymbol{r}_i}{dt} = \boldsymbol{V} + \boldsymbol{\Omega} \times (\boldsymbol{r}_i - \boldsymbol{R}) \qquad (7)$$

The force $\boldsymbol{f_i}$ acting on a boundary particle on the rigid body is due to the contact with other particles belong to different rigid bodies. This force can be calculated using any suitable contact model below.

2.3. Contact modelling for rigid bodies interaction

The interaction between boundary particles of blocks (Figure 1) are based on a concept of the spring and dash-pot system. This method is similar to that used in the DEM (Liu, 2018). Force-displacement laws proposed by Hertz's theory (Hertz, 1881) for the radial contact direction, no-slip elastic solution given by Mindlin (Mindlin, 1949) for the tangential direction. The radial force acting between two particles can be calculated using the following equation:

$$f_{a\to i}^{n} = \begin{cases} -K_{ai}\delta_n - c_n v_{ak}^{n} & h_{ai} > 2d_{ai} \\ 0 & h_{ai} \le 2d_{ai} \end{cases} \qquad (8)$$

where K_a is the radial stiffness, δ_n is the distance between two particles in the radial direction, c_n is the radial damping coefficient, v^n is is the relative radial velocity vector between particle *a* and *i*, h_a and h_i are the initial distance (smoothing length in SPH) between boundary particles in each rigid body, respectively, d_{ai} is the distance between two particles. The formulas calculated the shear stiffness coefficient, the relative displacement and the shear damping coefficient have been introduced by Cuong et al. (Cuong, 2013).

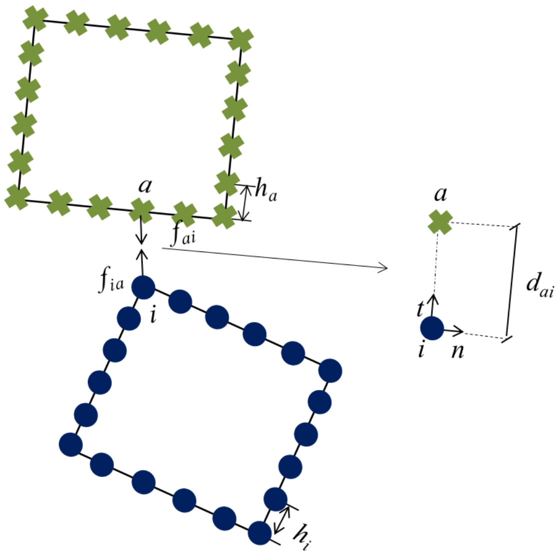

Figure 1. Schematic diagram of rigid bodies interaction

The contact force in the shear direction which is perpendicular to the radial direction can be calculated as Equation (9) and must satisfy Coulomb's friction law (C.T. Nguyen, 2013):

$$f_{a\to i}^{s} = \begin{cases} -k_{ai}\delta_s - c_s v_{ai}^{s} & h_{ai} > 2d_{ai} \\ 0 & h_{ai} \le 2d_{ai} \end{cases} \quad (9)$$

where k_{ai} is the shear stiffness, δ_s is the relative shear displacement between the two particles, c_s is the shear damping coefficient, v^s is the relative shear velocity vector between particle *a* and particle *i*. These parameters were determined by Cuong et al. (C.T. Nguyen, 2013)

2.4. Contact modelling between rigid bodies and hard ground

We consider a rectangular rigid body moves with initial velocity v_o, then the block enter in contact with hard ground (horizontal plane) and rebounds with a velocity v_x. At the time in contact, dissipation occurs and thus the body will rebound to a position which is lower than initial point. The formulas for falling and rebounding can be calculated:

$$h = \frac{1}{2}gt^2 \text{ and } h = \frac{(\sqrt{2gh_o}e^p - gt)^2}{2g} \quad (10)$$

where *h* is the height of the block at time *t*, *g* is the gravitational acceleration, *p* is the number of impacts, *e* is the coefficient of restitution can be obtained:

$$e = \frac{v_x}{v_o} \text{ or } e = \sqrt{\frac{h_x}{h_o}} \quad (11)$$

The maximum height of the body after each impact and the corresponding time interval are:

$$h_p = h_o e^{2p} \text{ and } t_p = 2e^p\sqrt{\frac{2h_o}{g}} \quad (12)$$

The effect of friction coefficient between the block surface and horizontaal plane on the motion of a rigid body. Based on the Newton's second law for the translational motion, when the block slides on a plane, thus the tangential force will be equal to the frictional force: $F = \mu N$ where μ is the friction coefficient between body surface and hard ground, *N* is the normal contact reaction. Accordingly, the decrease in velocity of the block may be given:

$$V = V_o - \frac{\mu N}{M}t \quad (13)$$

3. Results and discussion

3.1. Experimental model

To search the motion of rigid bodies and the interaction in movement progress due to gravity, the experimental setup is shown in Figure 2. The model ground is fixed during the conducted time.

Three rectangular-box structures are made from aluminum, each is numbers 1, 2 and 3, respectively. The aluminum rigid body is 32 mm in width, 25mm in height and 50 mm in length. The flat bar with two aluminum rods (the length can be changed) is used to stabilized the block from above. The experiment is constructed by successively placing one block on the top of the other with the overlapping value $\Delta x = 10$ mm. The physical properties of the blocks were expressed in Table 1.

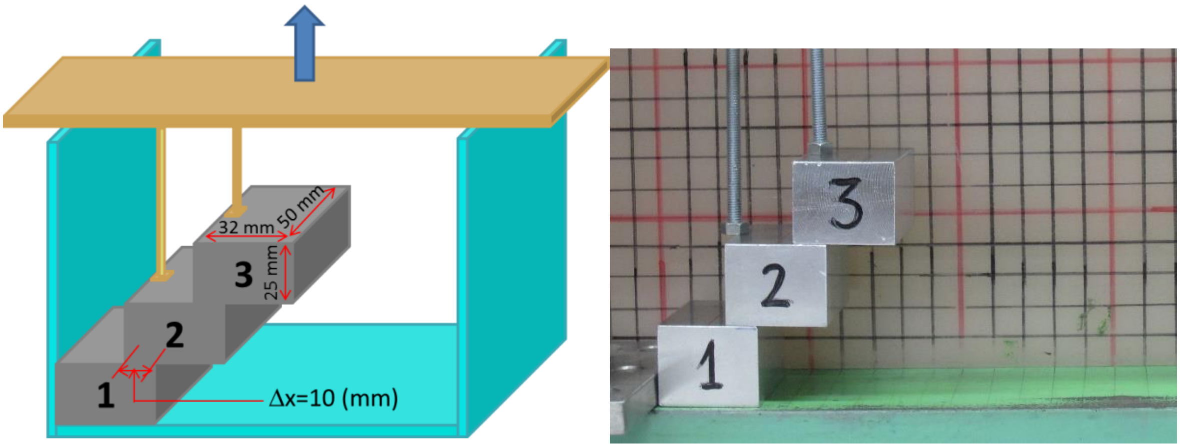

Figure 2. Experimmental model

The experiment were conducted by removing the flat bar and aluminum rods, so the blocks can move freely. The interaction between rigid bodies and between rigid bodies and plane surface with covering paint are proposed in the contact model. In addition, the friction coefficient (between aluminum block and model ground) is meassured k = 0.38 (dilatant angle $\alpha = 21^0$)

Table 1. Properties of structures

Name	Value	Unit
Density (ρ)	26.7	kN/m^3
Young's module (E)	69	GPa
Poisson's ratio (ν)	0.3	
Friction coefficient between rigid bodies (μ)	0.31	
Friction coefficient between rigid bodies and model ground (k)	0.38	

3.2. Experimental result

The experiment is repeated six times. The experimental result shown the position and rotation angle of the structures after moving on Figure 3 and the displacement of them (nearest left edge and farthest right edge) on Table 2.

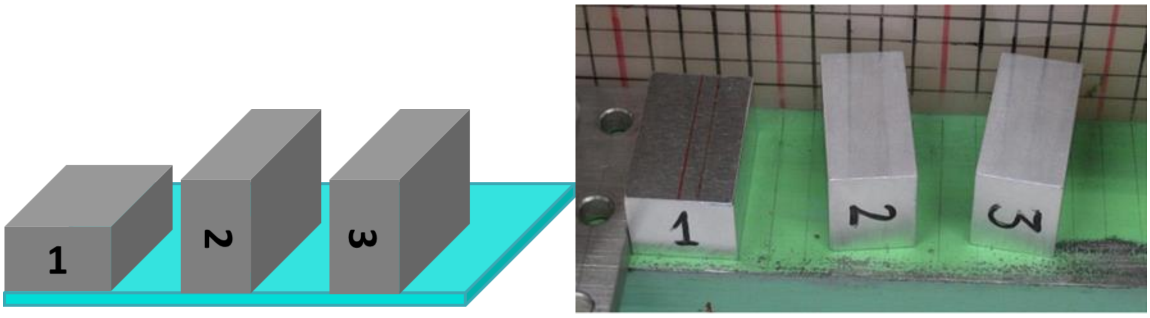

Figure 3. Position and rotation angle of the rigid bodies after moving

After moving. the rotation angle of these blocks are: Block 1 $\phi_1 \approx 0^0$. Block 2 $\phi_2 \approx 90^0$. Block 3 $\phi_3 \approx 90^0$.

Table 2. Position of blocks after moving

Number of repeat		1	2	3	4	5	6	**Average**
Left edge	Block 1	0	0	0	0	0	0	**0**
	Block 2	48.22	50.00	46.91	47.29	48.27	53.39	**49.01**
	Block 3	114.32	114.07	111.03	104.93	116.32	106.21	**111.15**
Right edge	Block 1	32	32	32	32	32	32	32
	Block 2	95.15	103.14	98.49	99.15	99.18	103.67	**99.8**
	Block 3	151.40	158.02	146.75	152.16	147.08	153.62	**151.51**

3.3. Numerical simulation

To compare the experimental result and numerical modeling in 2D problem, we identified the central position of block (average value of left point and right point) (Table 3). If this value falls within the displacement range of repeated six experiments, the numerical approach is absolutely suitable.

Table 3. Central position of blocks after moving

Number of repeat	1	2	3	4	5	6	**Average**
Block 1	16	16	16	16	16	16	**16**
Block 2	71.69	76.57	72.70	73.22	73.73	78.53	**74.41**
Block 3	132.86	136.05	128.89	128.55	131.70	129.92	**131.33**

In this experiment, we can see that the position of Block 1 doesn't change with the distance from the left-most boundary to the center of mass is 16 mm. Block 2 rotates an angle 90^0 and the central displacement (mean value) is 74.41 mm. Also, the rotation of Block 3 is 90^0 with the final run-out distance is 131.33 mm.

Figure 4 shows the movement in detail of the blocks in progress. Both Block 2 and Block 3 rotate 90^0 when they fall down the ground. After that, these blocks rebound and continue to enter in contact with the plane. While Block 2 begins to slide on the plane, Block 3 rebounds more than moving on the right side. So, the same rotation can be seen in the experiment and simulation.

The final resting distance between the left boundary of the wall and the central position of Block 2 in the computed result turns out to be 76.05 mm (Figure 9k), whereas in the experiment, this distance is measured at 74.41 mm. This result definitely falls within the distance range observed in repeated experiments (the smallest and the greatest value, respectively, are 71.69 mm, 78.53 mm) (Table 3).

It can be seen that the central position of Block 3 in simulation is approximately 130 mm. This result is in fairly well agreement with that mean value observed in the experiment is 131.33 mm. Thus, the calculation is good agreement between the minimum and the maximum value in Table 3 (128.55 mm and 136.05 mm).

Consequently, the numerical result of the proposed model is suitable for the modeling block's state. The rotation of blocks between the experiment and simulation is the same. Moreover, the final run-out position in computing fall within the value domain of the experiment.

On the whole, the strong point of the method is that the rotation motion of rectangular-box structures, which the traditional numerical approaches are impossible or difficult to simulate. Besides, the considerable potential is also offered for modelling interaction in the different environments.

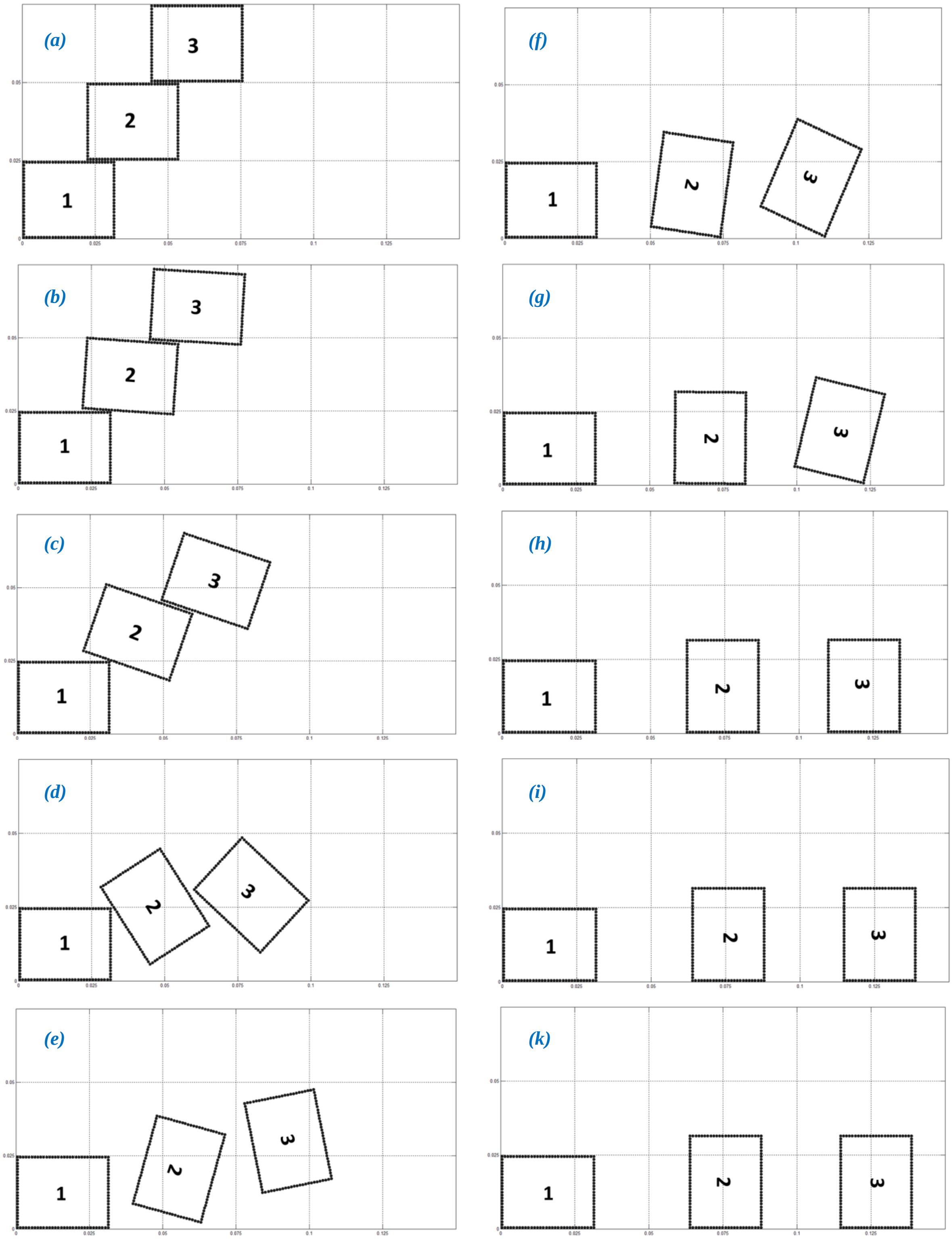

Figure 4. Motion of rigid-bodies in numerical simulation

4. Conclusion

This research showed in detail interaction and movement behaviours of rectangular-box rigid bodies by physical experiments. These results are very useful for other studies to compare the experimental problems and verify a numerical model.

The Smoothed Particle Hydrodynamics (SPH) method was used in the computing program based on Fortran 90. This numerical approach could simulate well the behaviours in both traditional and rotational movement of 2D structures. It is shown that the proposed tool should be considered to research stabilize the building construction having rectangular-box structures, like the retaining wall system, dam, etc.

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